Time, Frequency and Time-Frequency Domain Methods for Dielectric Parameter Identification

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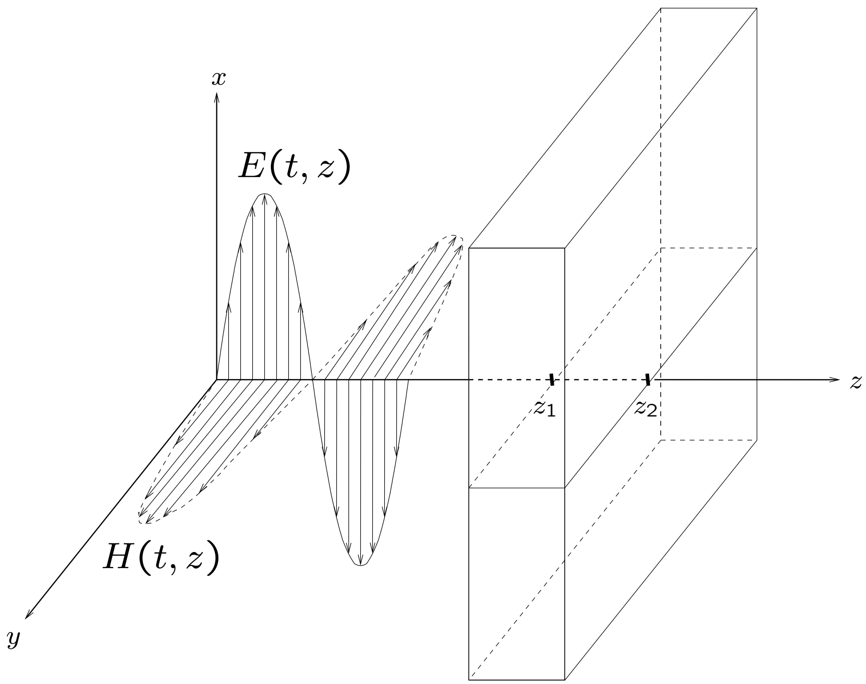
Outline:

1. Physical Problem and Identification Problem

2. Time, Frequency and Time-Frequency Domain Objectives

3. Methods and Results

Physical Problem:



- Planar EM wave normally incident to dielectric slab.
- EM wave: windowed microwave signal.
- Reflection of wave off slab interfaces is observed.
- Properties of slab described by polarization equation

Maxwell's Equations:

$$\begin{aligned} \nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H} &= \frac{\partial\vec{D}}{\partial t}+\vec{J} \\ \nabla\cdot\vec{D} &= \rho \\ \nabla\cdot\vec{B} &= 0. \end{aligned}$$

$$\begin{aligned} \vec{D} &= \epsilon_0\vec{E}+\vec{P} \\ \vec{B} &= \mu_0\vec{H}+\mu_0\vec{M} \\ \vec{J} &= \vec{J}_c+\vec{J}_s \end{aligned}$$

Quantities $\vec{P},\vec{M},\vec{J}$ describe material behavior, require constitutive laws to complete. We use: $\vec{M}=0,\vec{J}=\sigma\vec{E}$.

Debye Polarization Model:

$$\begin{aligned} \tau \dot{\vec{P}} + \vec{P} &= \epsilon_0 (\epsilon_s - \epsilon_\infty) \vec{E} \\ \vec{D} &= \epsilon_\infty \epsilon_0 \vec{E} + \vec{P}. \end{aligned}$$

First order differential equation for $\vec{P}$ models permanent dipole relaxation.

Modification of $\vec{D}$ equation allows for an instantaneous component of polarization.

Higher order, and integral models possible as well.

Interrogation Simulation:

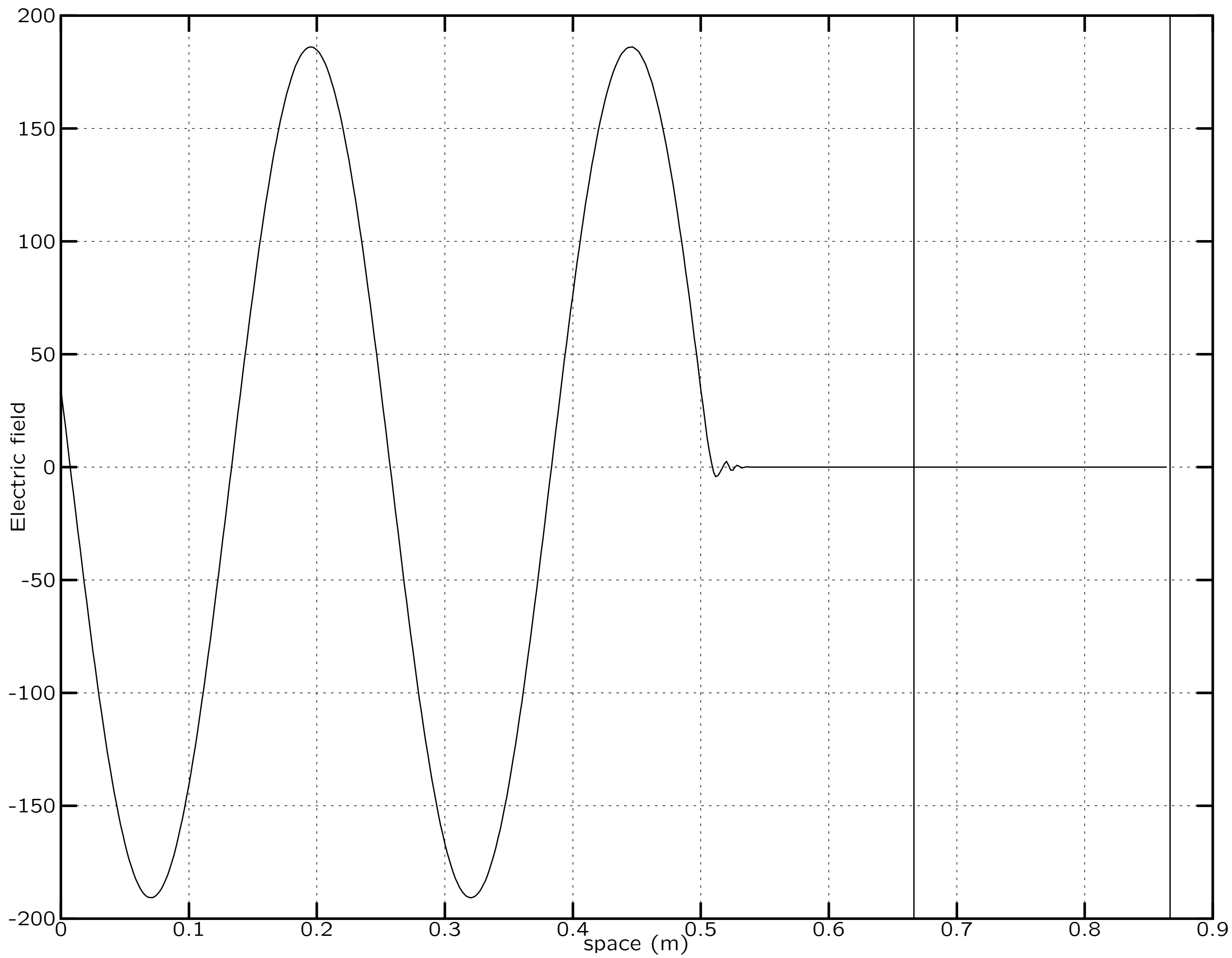
- Current source $\vec{J}(t)$ placed at $z = 0$. Symmetry makes problem one-dimensional. $\vec{E} = \widehat{x}E(z,t)$,$\vec{H} = \widehat{y}H(z,t)$. Slab occupies $(z_1, z_1 + d)$.

- Parameters $(\sigma, \epsilon_\infty, \epsilon_s, \tau, d)$ chosen for the material slab.

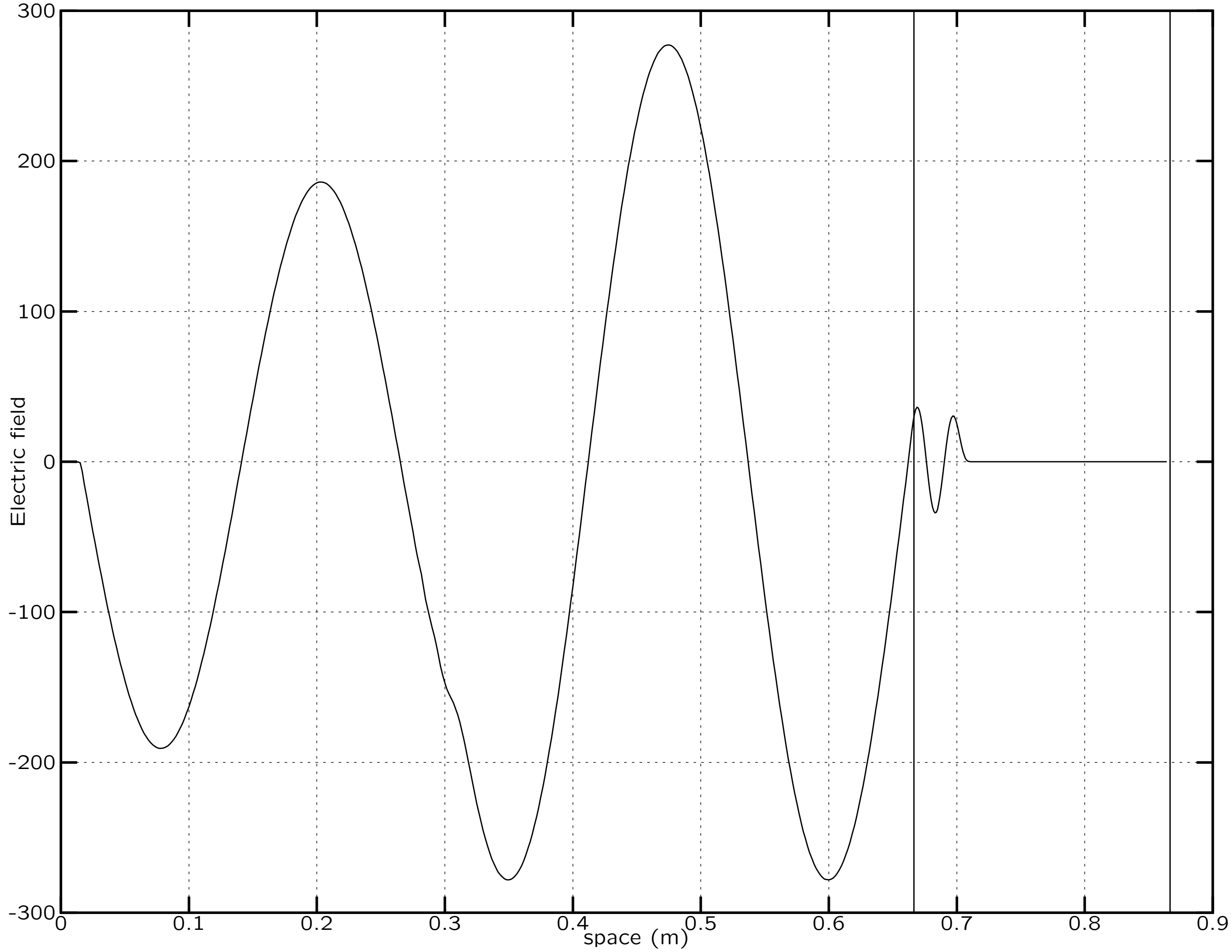
- A particular form for the current source is chosen.

- Observations taken at $z = 0$ show original signal, reflection off z_1 interface, then reflection off z_2 interface.

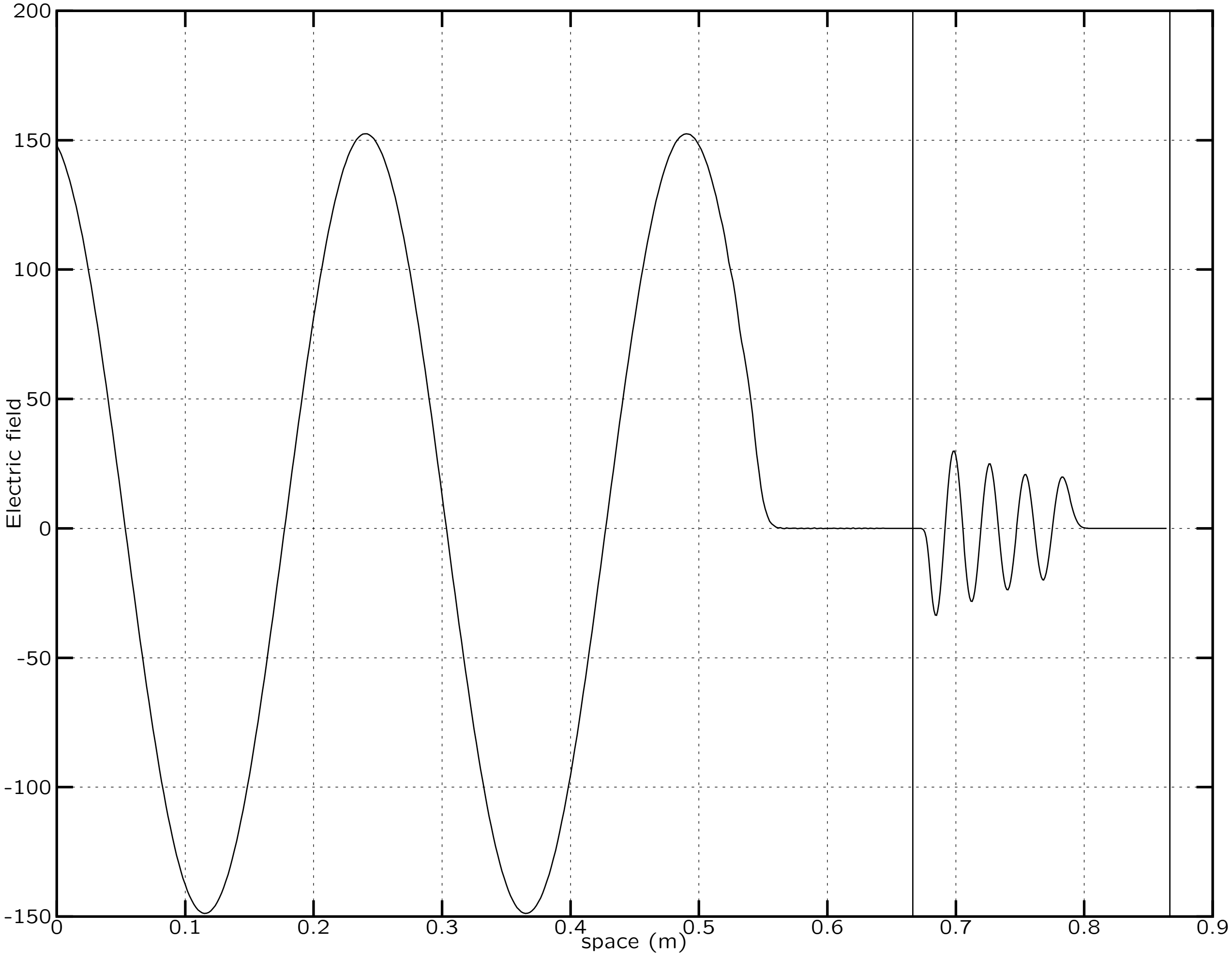
Forward Problem Example: $t = 1.7$ ns



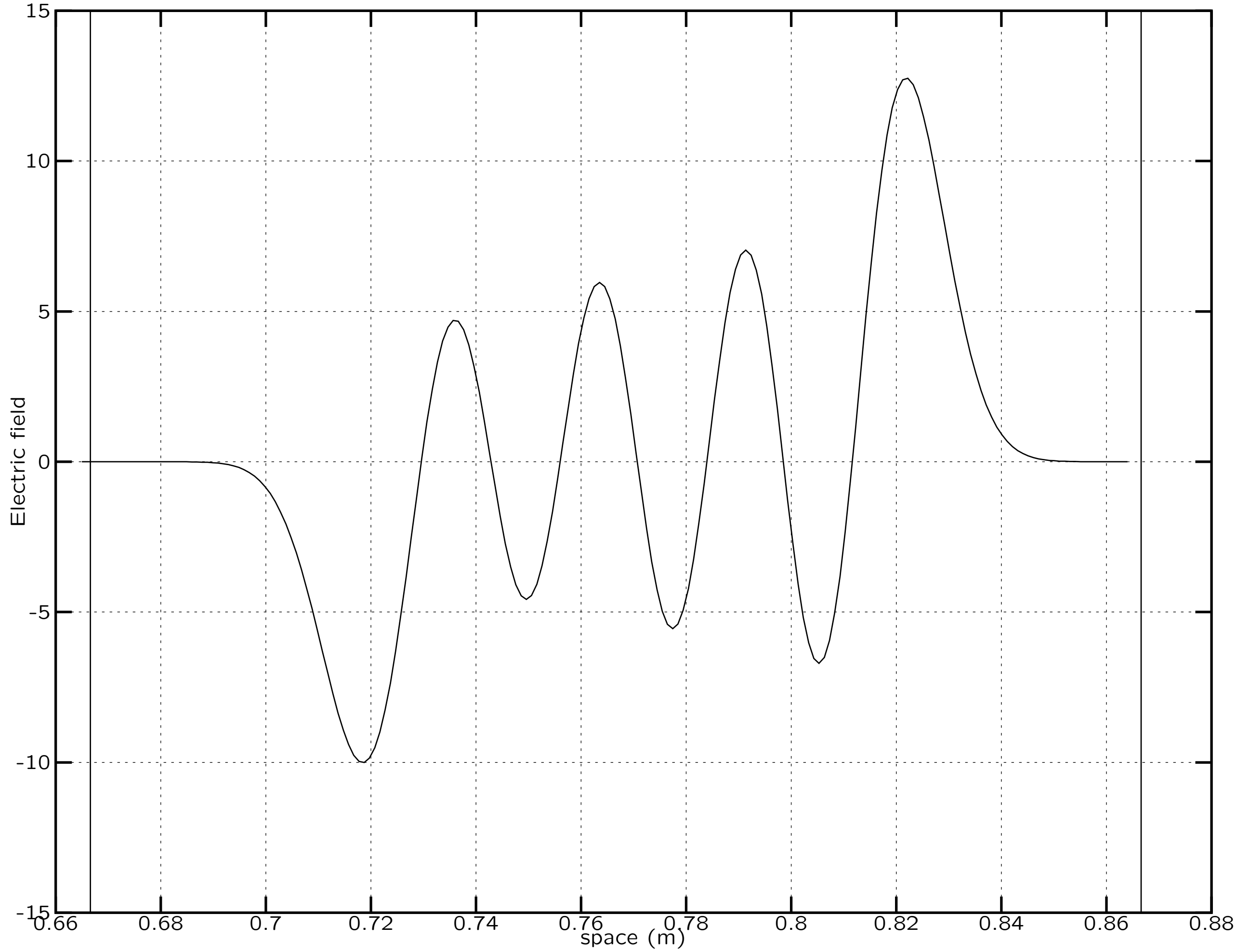
Forward Problem Example: $t = 3.4$ ns



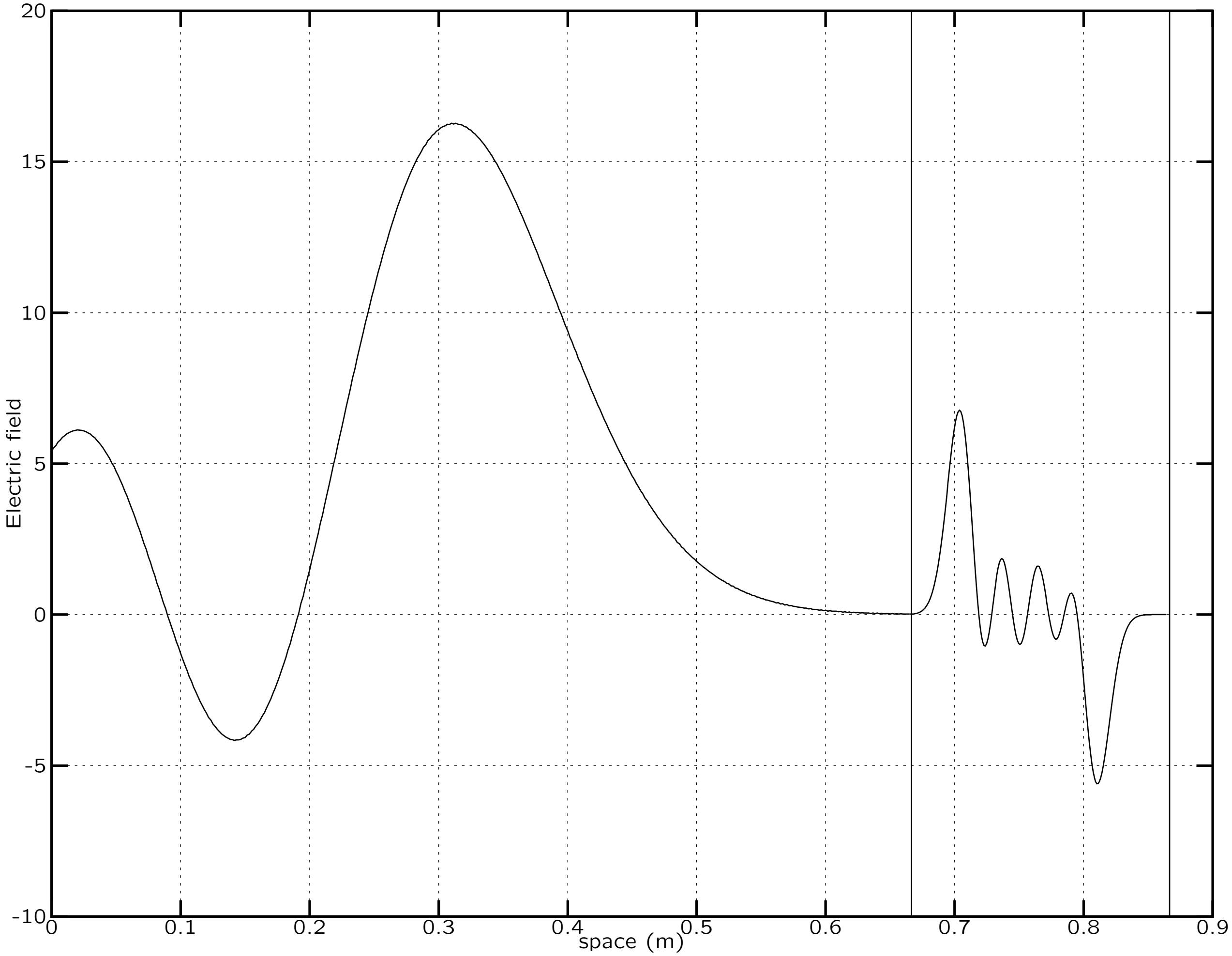
Forward Problem Example: $t = 5.75$ ns



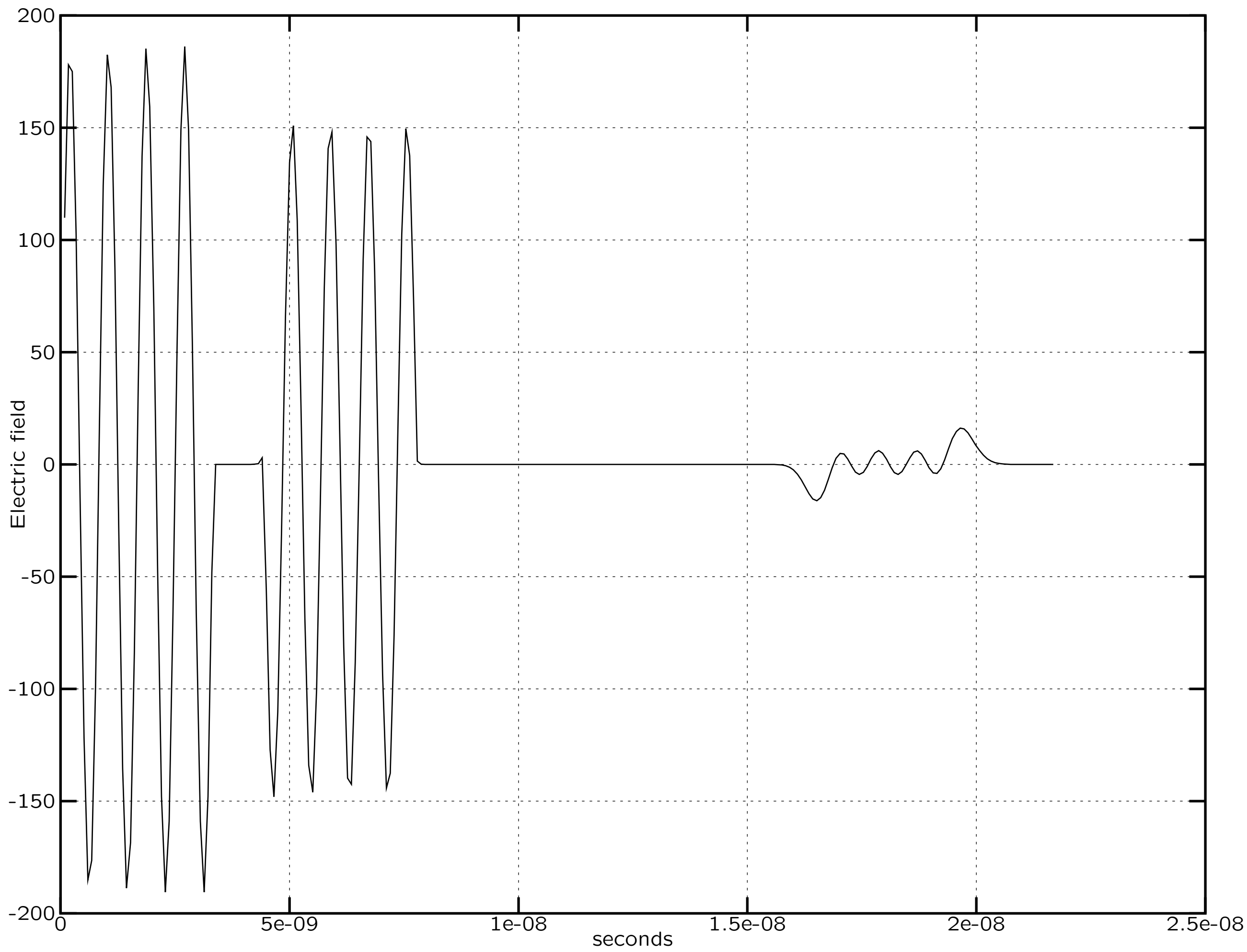
Forward Problem Example: $t = 12.75$ ns



Forward Problem Example: $t = 18.7$ ns



Forward Problem Example: $z = 0$ meters



Parameter Values:

We use the following parameter values for our examples.

σ	0.0	mhos/meter
ϵ_S	81.1	
ϵ_∞	5.5	
τ	8.1×10^{-12}	seconds

Except for σ, these are the accepted values for water.

Interrogating Signal:

Carrier frequency $f \approx 0.6$ GHz to 2.4 GHz or an increasing frequency chirp.

Windowed with rectangular or gaussian pulse.

Identification Problem:

Given observations of the electric field at $z = 0$: $E_i = E(0, i\Delta t)$, find parameters $(\sigma, \epsilon_\infty, \epsilon_s, \tau, d)$ which give the best fit to the data.

"Best Fit" is determined by various criteria. Generally a least-squares distance between various transformations of the time-domain data.

Previous Results:

Time-domain identification of Debye model parameters.

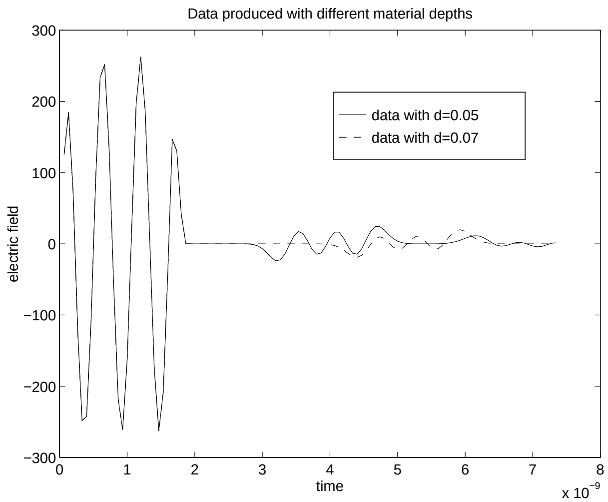
Accurate reconstruction of $\epsilon_s, \epsilon_\infty, \tau$ in presence of moderate noise in signal. ($< 5\%$).

Reconstruction of σ diffucut for reasonable values. ($\times 10^{-5}$)

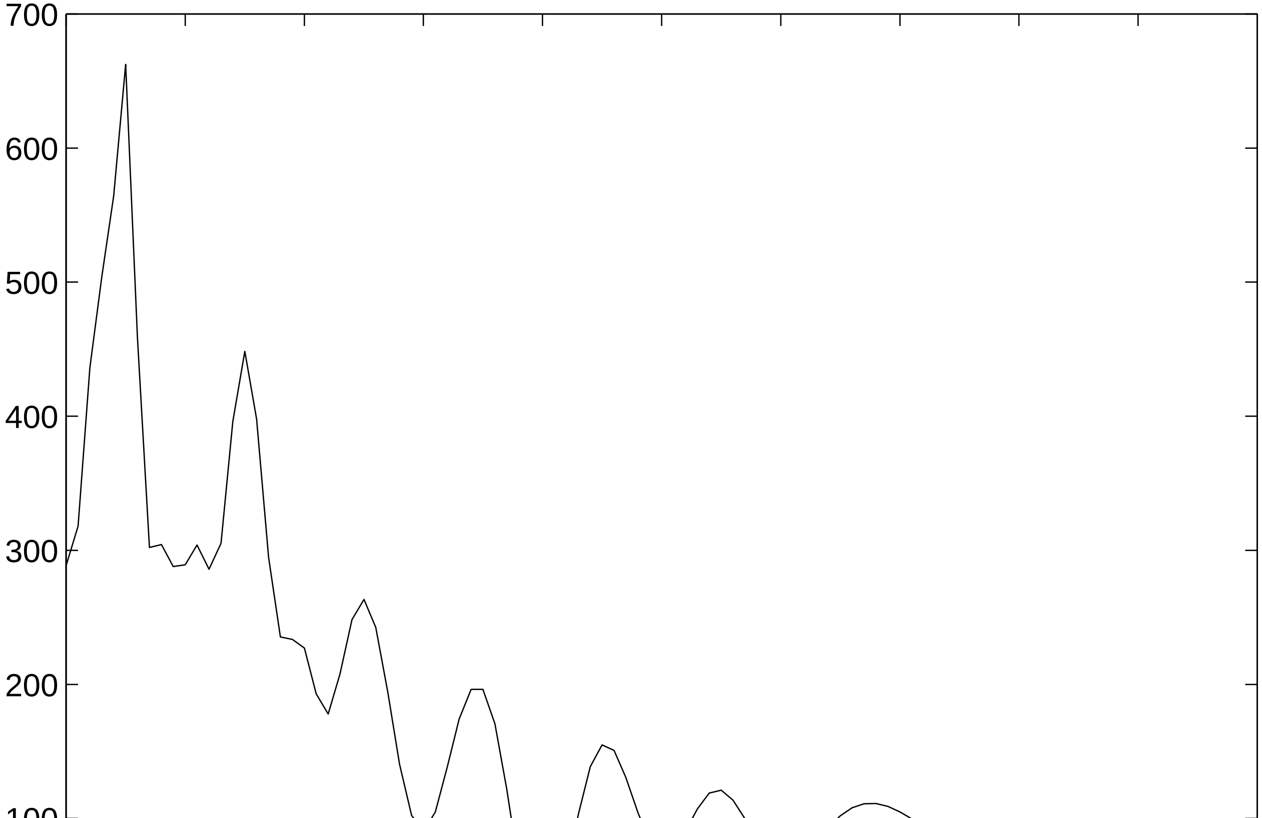
Easier to identify $\tau(\epsilon_s - \epsilon_\infty)$ than ϵ_∞.

Difficulty with depth estimation lead to three step identification process:

Effect of incorrect depth:



Time-domain Error as a function of depth:


700
600
500
400
300
200
100

Three Step Method:

Identify $(\tau, \epsilon_S, \epsilon_\infty)$ with surface reflection. Identify d through return time of deep reflection. Use as initial guess to estimate $(\tau, \epsilon_S, \epsilon_\infty, d)$ to refine estimates of all parameters.

New Distance Measures:

Distance measures based on frequency and time–frequency transformations of the original and simulated data.

Frequency transform: absolute value of DFT of signal.

Time-Frequency Transform:

The Fourier Transform

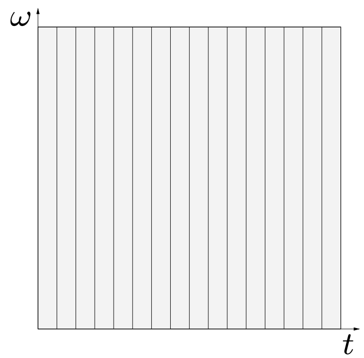
$$\widehat{f}(\omega) = (\mathcal{F}f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-\imath\omega t}\, dt$$

The Short Time Fourier Transform

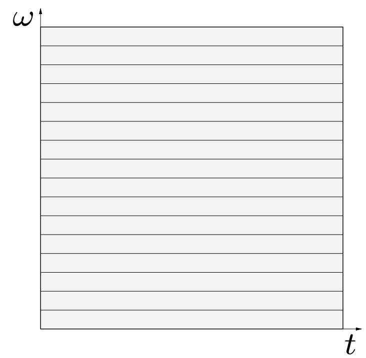
$$(\mathcal{F}_w f)(\omega, \tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) w(t-\tau) e^{-\imath\omega t}\, dt$$

w a windowing function such as the Gaussian $w(t) = e^{-t^2}$.

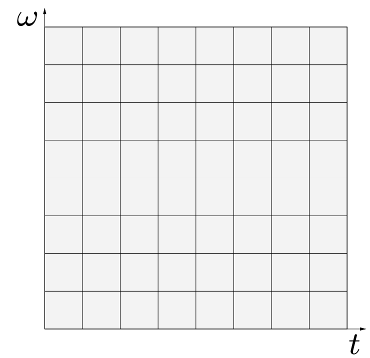
Time-frequency resolution of standard function representations.



Time:
$f(t)$



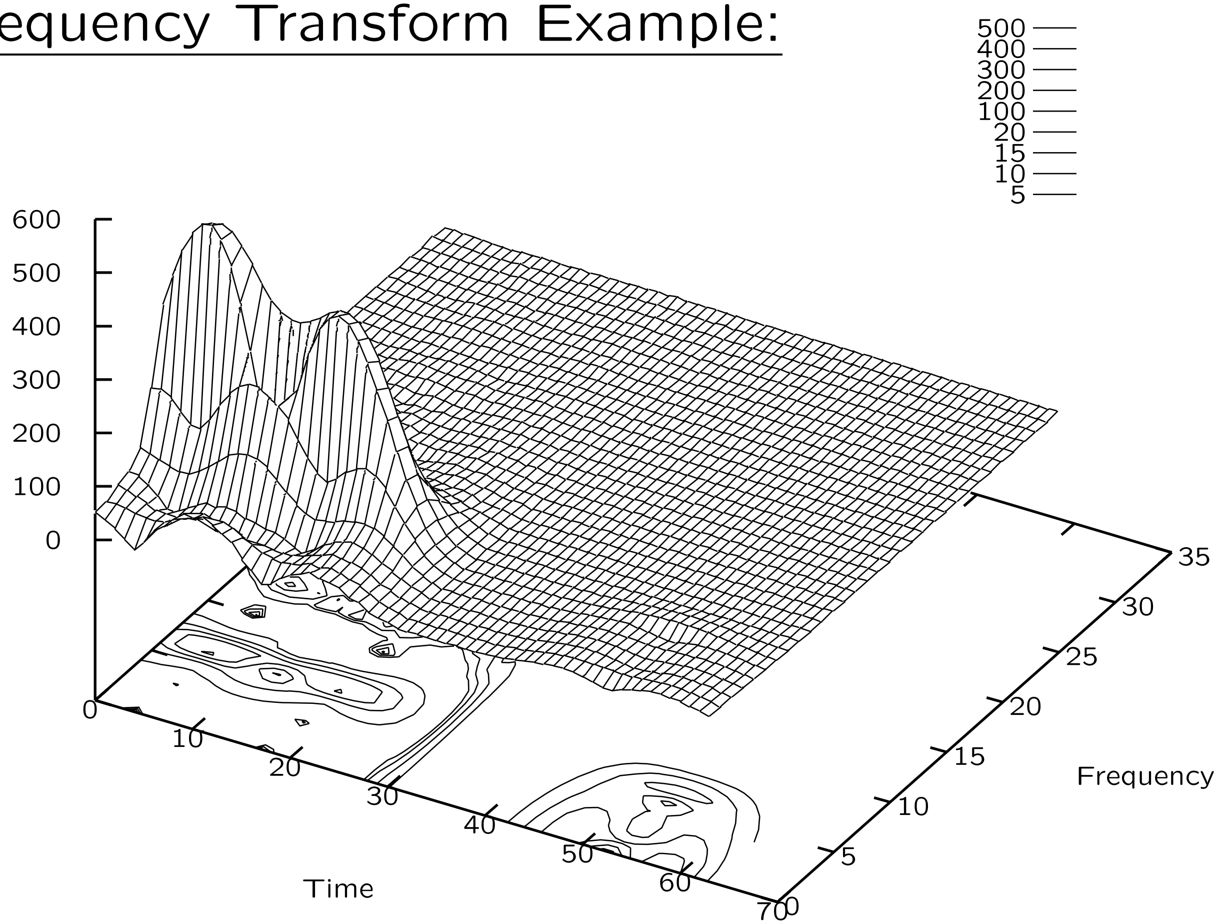
Frequency:
$(\mathcal{F}f)(\omega)$



Time-frequency:
$(\mathcal{F}_w f)(\omega, \tau)$

$(\mathcal{F}_w f)(\omega, \tau)$ provides a measure of the amplitude of frequency ω at time τ.

Time-Frequency Transform Example:



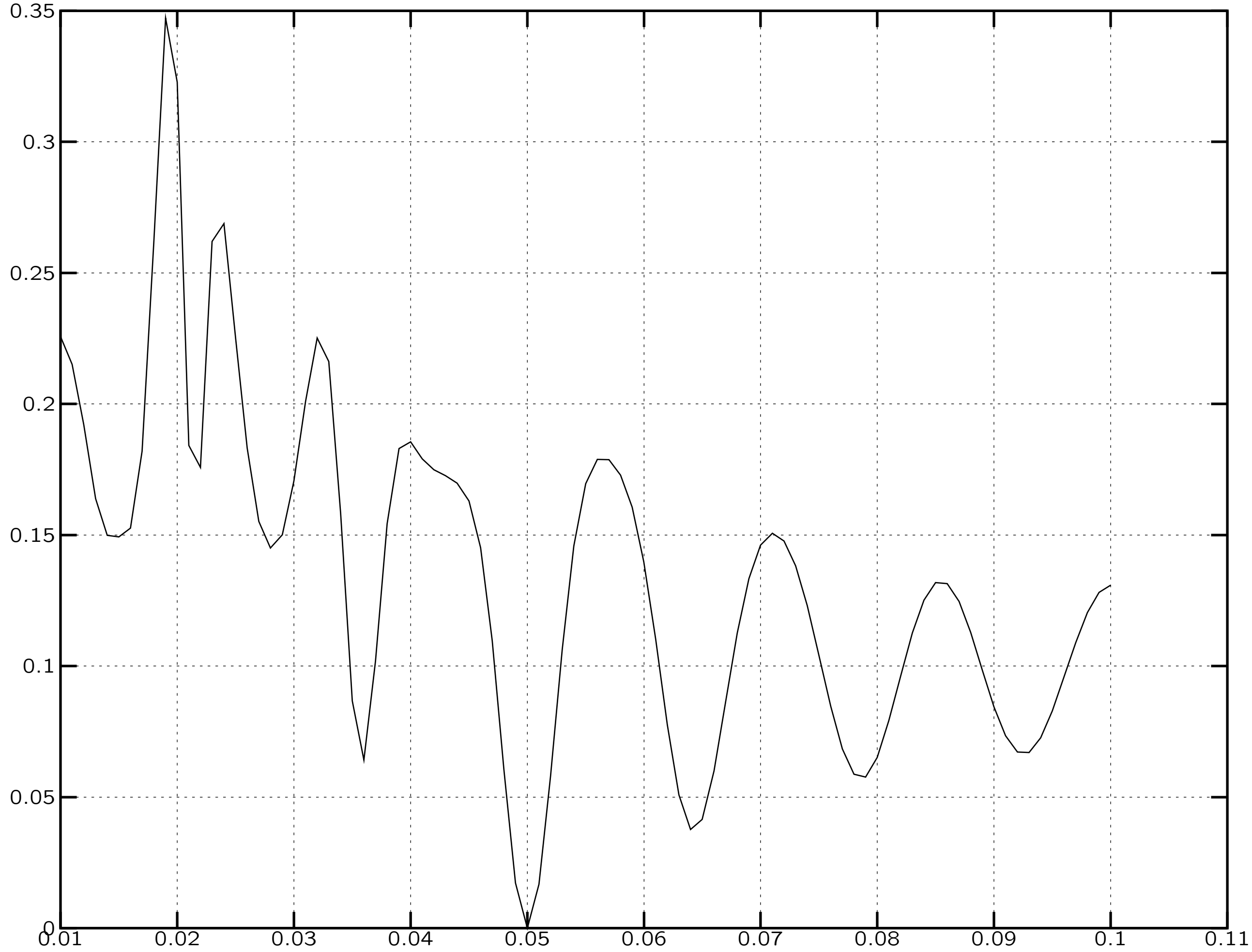
Implementation Details:

Forward simulations done with FE-TD scheme: PW-linear elemets in space, Crank-Nicholson scheme in time. Implemented in Fortran.

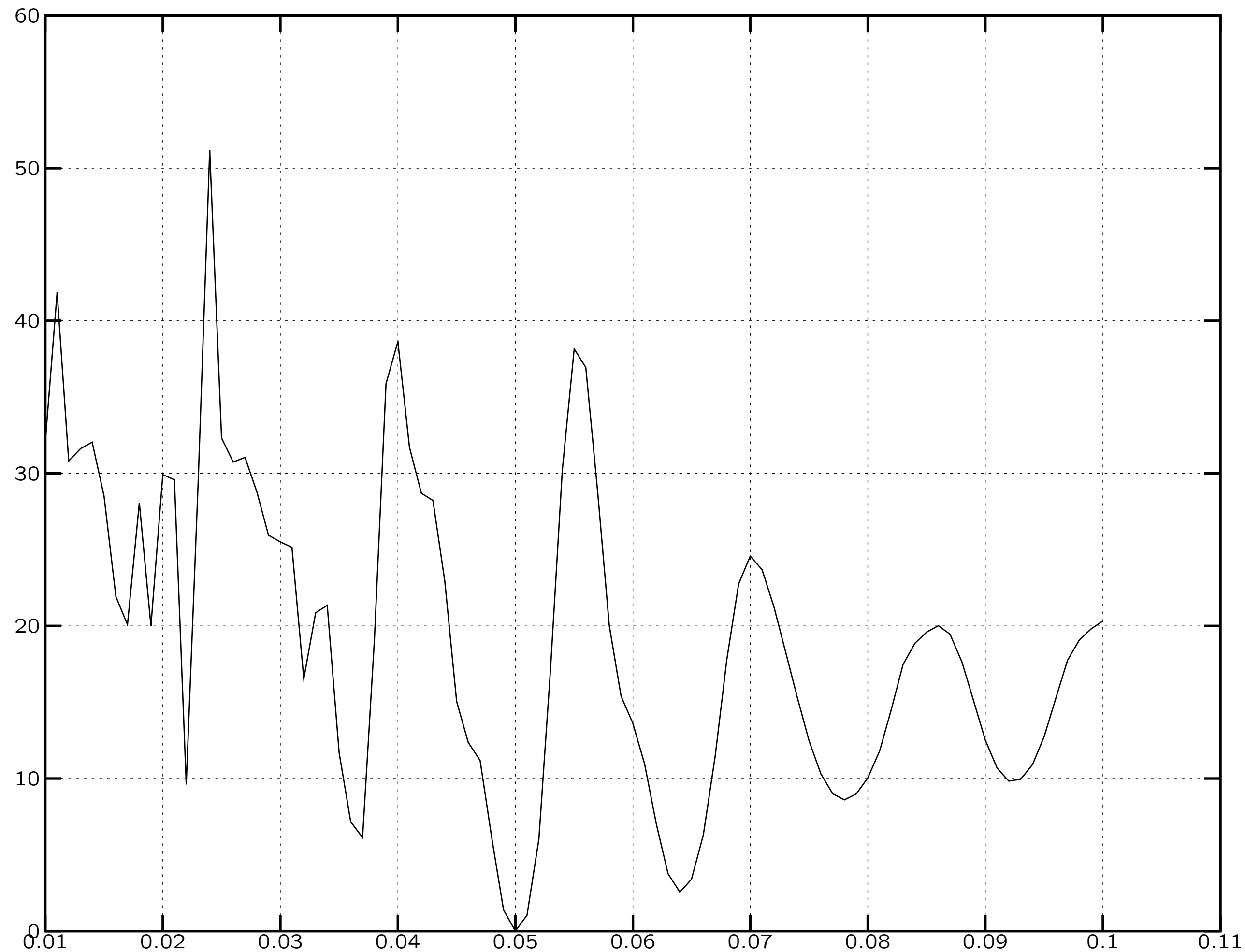
Optimization handled by BFGS/Trust region code (Dr. Carter, ICASE) in Fortran.

Transforms, distance functions, data processing routines are written as Octave scripts (free Matlab clone) with interfaces to Fortran codes written in C++.

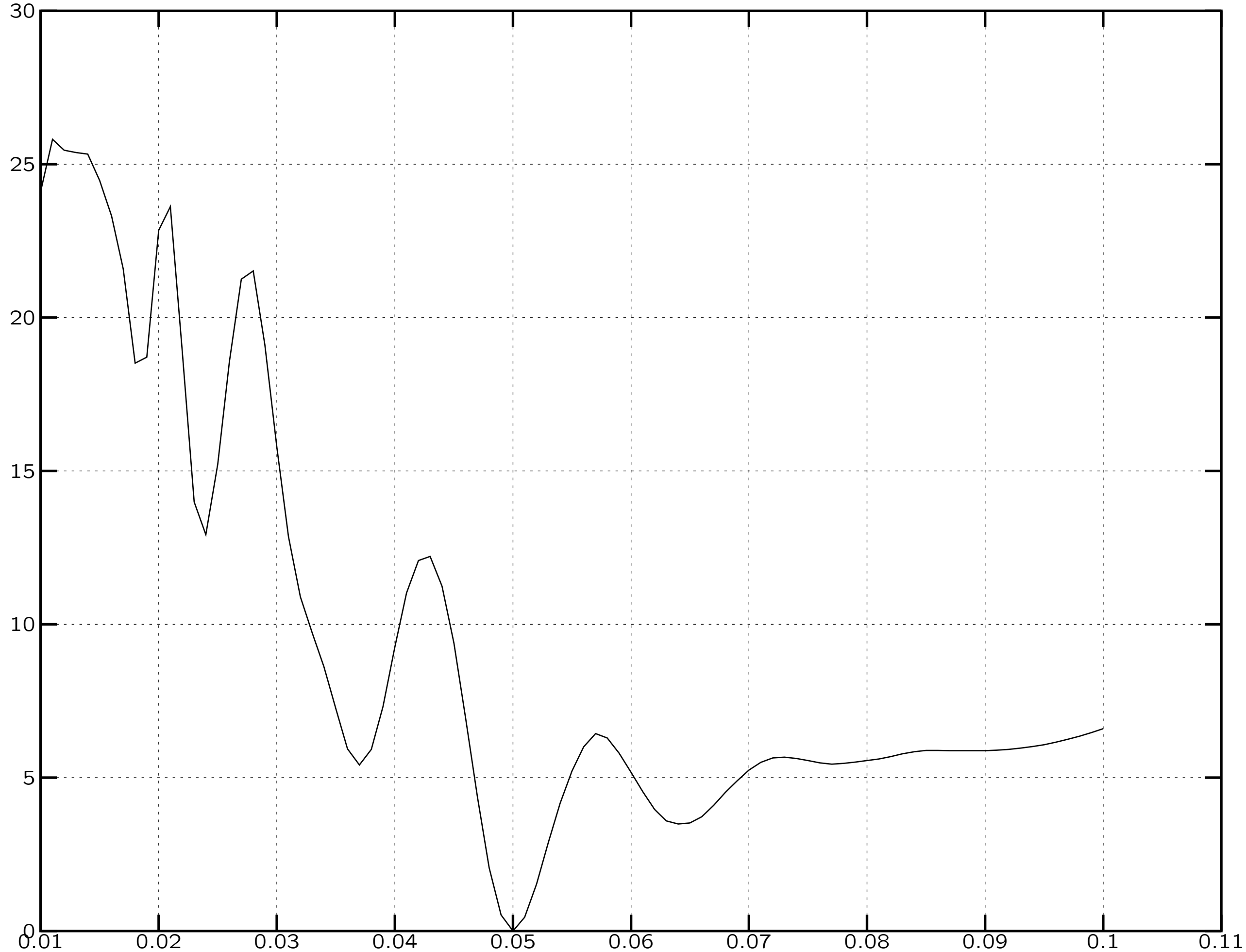
$J(d)$ in time domain for rectangular pulse



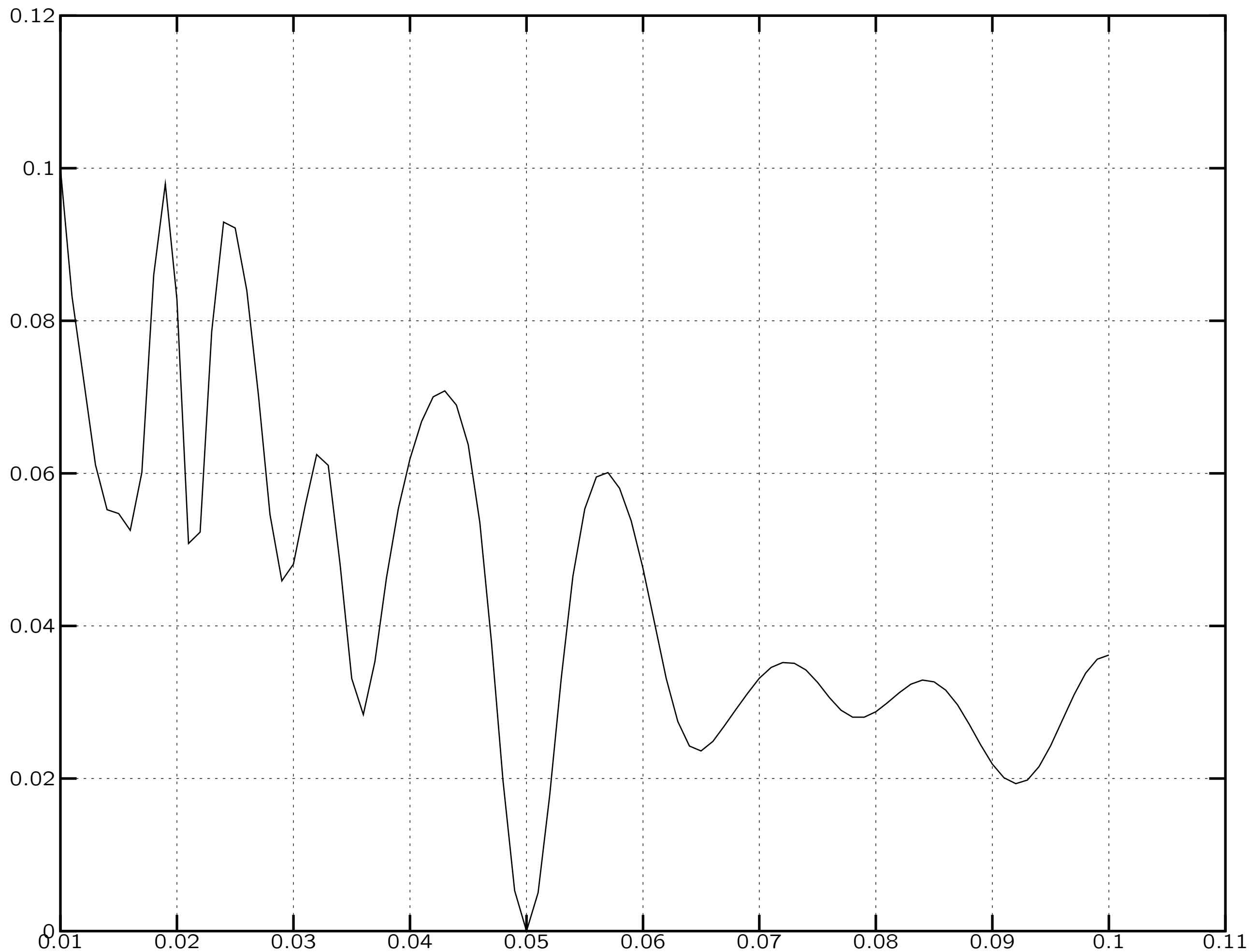
$J(d)$ in frequency domain for rectangular pulse



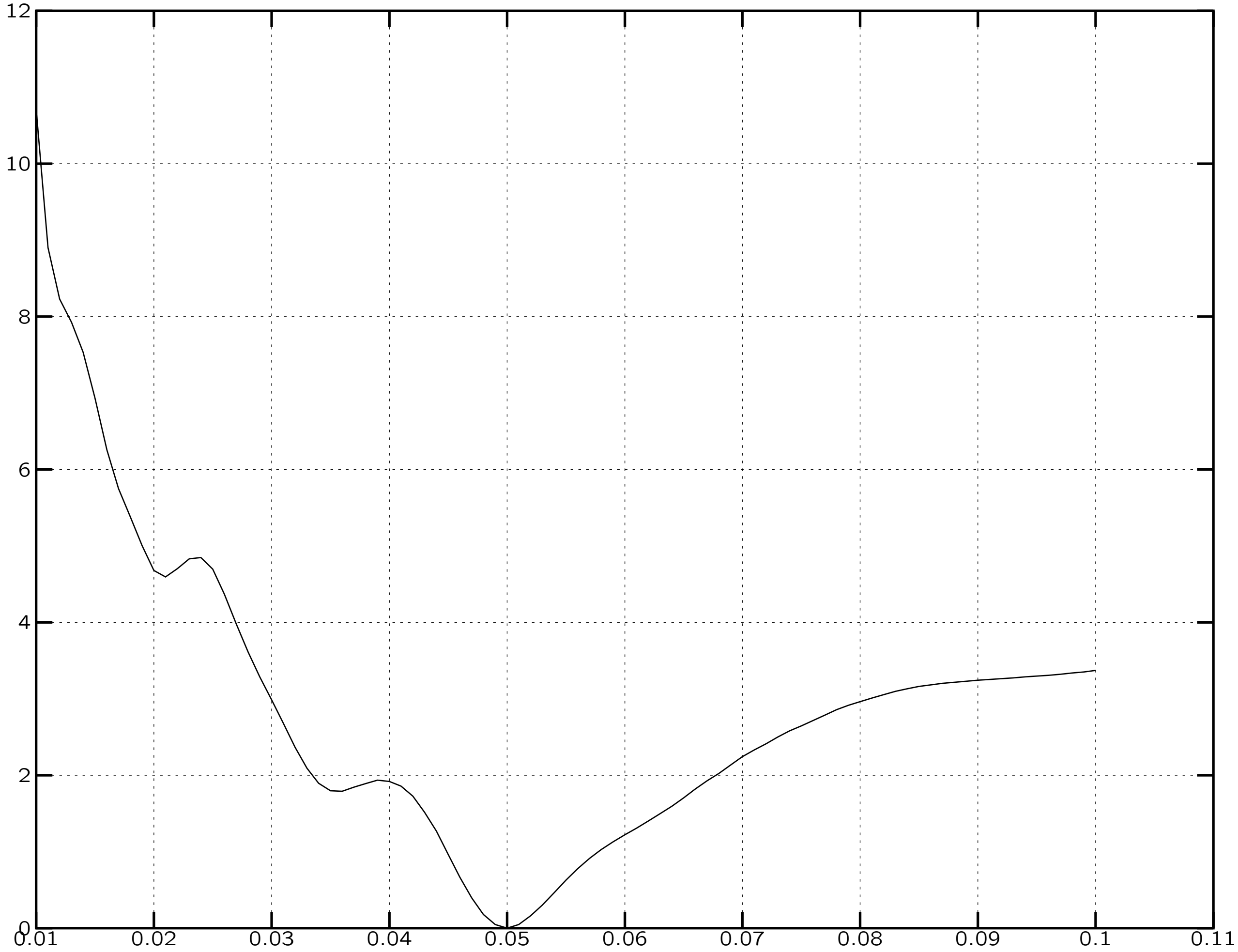
$J(d)$ in time–frequency domain for rectangular pulse



$J(d)$ in time domain for gaussian chirp



$J(d)$ in time–frequency domain for gaussian chirp



Tested:

Four input signals. 40 initial parameter sets. Time and Time-frequency distance measures. (160 runs each)

Results:

	Time	Time-Frequency
Converged	100	76
Improved on Initial Iterate	24	14
Close to Correct Parameters	11	5

Future Work:

- Explore time-frequency transformation options to find more effective smoothers of the objective function.

- Determine optimal objective criteria and input signal combinations for various identification problems

- Add Noise to signal. Estimate noise tolerance of inverse problem for various objective functions and signal types.